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COMMENT

Simulation of the Compton effect by reflection from a moving mirror

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Abstract. An examination is made of Ashworth's analogy between the Compton effect and the reflection of a photon from a moving mirror. It is shown that one of his allowed mirror orientations does not succeed in simulating the Compton effect and that the correct result may be obtained by a much simpler argument.

Recently Ashworth (1978) has pointed out that the energy and direction of motion of the scattered photon in a Compton scattering process are the same as would be produced by allowing a photon with the same initial energy and direction of motion to be reflected from a moving mirror. Ashworth's analysis yields a single value for the velocity of the mirror, but two distinct solutions for the angle α which specifies its orientation. The purpose of this note is to show that one of these solutions is spurious, being due to an ambiguity in the sign of the angle used in Ashworth's method to define the direction of the reflected photon. Furthermore we show how the correct result may be obtained by a much simpler procedure and that it applies to a particle–particle collision as well as to a particle–photon collision.

We first set out our own analysis for a particle–particle collision. Consider a particle of rest mass M_0 which is initially at rest in an inertial frame S and which moves after the collision with uniform velocity V in the positive x direction. The second particle initially has energy E_1 and moves with momentum p_1 in a straight line making an angle ψ with the positive x axis. After collision it has energy E_2 and moves with momentum p_2 in a straight line making an angle $\psi+\phi$ with the positive x axis. The geometry is as indicated in figure 1 of Ashworth's paper. Conservation of energy and momentum requires

$$E_1+M_0c^2=E_2+\gamma(V)M_0c^2, \tag{1}$$

$$p_1\cos\psi=\gamma(V)M_0V+p_2\cos(\psi+\phi), \tag{2}$$

$$p_1\sin\psi=p_2\sin(\psi+\phi), \tag{3}$$

where $\gamma(V)=(1-V^2/c^2)^{-1/2}$.

We now enquire whether there is a frame S′, moving with respect to S with velocity v in the x direction, in which the initial and final energies of the second particle are the same, since this is one of the conditions necessary for the behaviour of this particle to be represented as 'specular reflection'. Taking S and S′ to be in standard configuration, we apply the energy transformation equation

$$E'=\gamma(v)(E-vp_x) \tag{4}$$

to the second particle before and after the collision, and find on equating E_1' and E_2' that

$$E_2 - vp_2 \cos(\psi + \phi) = E_1 - vp_1 \cos \psi. \tag{5}$$

Combining equations (1), (2) and (5) we quickly discover (in agreement with Ashworth) that

$$v = (c^2/V)[1 - (1 - V^2/c^2)^{1/2}]. \tag{6}$$

As noted by Ashworth, in the frame S′ the first particle moves with speed v in the negative x' direction before the collision, and with speed v in the positive x' direction afterwards. In this frame it follows from conservation of momentum together with $E_1' = E_2'$ that the x' component of momentum for the second particle is also reversed by the collision, the y' component of momentum being unaffected by the collision. The motion of this particle can therefore be represented as specular reflection from a 'particle mirror' which is at rest in S′ and whose normal is parallel to the x' axis. This orientation is the $\alpha = 0$ solution obtained by Ashworth. Clearly the analysis for a particle–photon collision is almost identical, only the minor substitutions $E_1 = h\nu_1$, $p_1 = h\nu_1/c$, etc being required to re-express the argument in terms of photon frequencies. The conclusion is the same, except of course that the 'particle mirror' may now be regarded as an ordinary (optical) mirror.

We now comment on the second solution for α found by Ashworth. His procedure involved the use of previously derived formulae (Ashworth and Davies 1976) for the frequency ν_2 and direction ϕ_2 of a light ray reflected by a mirror which moves in S with constant velocity v in the x direction and whose normal makes an angle α with the x axis, the initial frequency and direction of the light ray being ν_1 and ϕ_1 respectively (ϕ_1 and ϕ_2 are defined with respect to the positive x axis.) Ashworth equated ν_2/ν_1 to the corresponding expression obtained from Compton effect analysis, and equated $\cos \phi_2$ to the expression for $\cos(\psi + \phi)$ in the Compton effect, then solved the resulting simultaneous equations for v and α. However, a spurious second solution for α arises in this method from the fact that $\cos \phi_2 = \cos(\psi + \phi)$ has two solutions for ϕ_2, i.e. $\phi_2 = \psi + \phi$ (the solution required) and $\phi_2 = -(\psi + \phi)$ (an irrelevant solution). In other words, the procedure adopted unfortunately solves not only the problem of finding a mirror which gives the same reflection parameters ν_2, $\psi + \phi$ as occur in the Compton effect, but also the problem of finding a mirror which gives the reflection parameters ν_2, $-(\psi + \phi)$, a problem of comparatively little interest. This is largely confirmed by Ashworth's figure 3, which attempts to portray the geometrical arrangement for the second solution for α; if the mirror is replaced by the original electron moving parallel to the x' axis throughout, then it is obvious (especially in S′) that momentum is not conserved.

Finally we would like to comment briefly on the significance which Ashworth attaches to this electron–mirror analogy. It is admittedly true that 'the scattering electron in the Compton effect is therefore acting like a perfectly reflecting mirror . . .', but we believe that there is a danger in pressing the analogy too far, e.g. in speculating on '. . . the possibility that the Compton effect might be an example of light being reflected from a perfectly reflecting moving mirror . . .'. As we have noted, specular reflection occurs in the frame S′ not only for a particle–photon collision but also for a particle–particle collision, and this feature of relativistic particle dynamics is therefore not something peculiar to the Compton effect or, in our view, likely to reveal anything about its nature. The essential non-classical feature in the conventional explanation of the Compton effect lies in the corpuscular nature of the radiation expressed by the equations $E = h\nu$, $p = h\nu/c$, and nothing in Ashworth's analysis appears to suggest that the Compton effect might be explicable in classical terms not involving the photon hypothesis.

References

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